Loss of entanglement in quantum mechanics due to the use of realistic measuring rods

Rodolfo Gambini
Instituto de Física, Facultad de Ciencias, Universidad de la República, Iguá 4225, CP 11400 Montevideo, Uruguay

Rafael A. Porto*
Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213

Jorge Pullin
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803-4001
(Dated: September 19th. 2007)

We show that the use of real measuring rods in quantum mechanics places a fundamental gravitational limit to the level of entanglement that one can ultimately achieve in quantum systems. The result can be seen as a direct consequence of the fundamental gravitational limitations in the measurements of length and time in realistic physical systems. The effect may have implications for long distance teleportation and the measurement problem in quantum mechanics.

arXiv:0708.2935v2 [quant-ph] 6 Feb 2008

Quantum field theory is ordinarily formulated in terms of wavefunctions $\Psi(\vec{x},t)$ where $\vec{x}$ and t are assumed to be classical parameters that can be measured with arbitrary accuracy. In reality, $\vec{x}$ and t will be determined through the measurement of physical quantities that correspond to operators. Therefore they will have uncertainties in their measurement. In a series of recent papers [1] we have explored this issue, in particular using fundamental limits on the level of accuracy of measurement of distances and times. Surprisingly, the fundamental limits are gravitational in origin, since to measure things with very high accuracy one requires large amounts of energy and therefore gravity becomes relevant. As an example of the type of effects encountered, in quantum mechanics formulated with real clocks, evolution is not unitary, since as time evolves the real clocks will fail to mirror exactly the evolution of the parameter t that appears in the Schrödinger equation [2]. In quantum field theory in addition to this effect one has others since both time and space must be measured using real devices [3]. In this paper we would like to discuss another effect that arises due to the use of real clocks and rods in quantum theory: that one cannot achieve the maximum level of entanglement in a quantum system. Entanglement is a fundamental effect of quantum mechanics through which a composite system manifests non-local correlations in space. A maximally entangled state is one in which one obtains the maximum violation possible for a system of Bell inequalities. The latter can be considered as the limit of correlations achievable by systems that behave classically and locally. We will show that the level of entanglement that one can achieve in a system is less than the usual quantum mechanical limit due to limitations in our ability to measure distances, and that the effect increases with the distance between the components of the system, and also with respect to the distance to the observer.

In order to illustrate this issue we consider a non relativistic electron field. We will assume we have a detector that can measure the spin of the electron field within a certain region $v_{\vec{x}}$. The detector is placed at some coordinate position $\vec{x}$ and we assume the region $v_{\vec{x}}$ is centered at $\vec{x}$. We do not have direct access to the value of $\vec{x}$ but we assume we have a system of real measuring rods that assign values represented by a quantum field $\vec{X}(\vec{x})$. This assigns a value for the position of certain reference point of the detector as a function of its fiducial unobservable coordinates $\vec{x}$. We then set up an observable that measures the spin, $\hat{\sigma}^z(v_{\vec{x}})$, given by

$$\hat{\sigma}^z(v_{\vec{x}}) \equiv \frac{1}{2}\int_{v_{\vec{x}}} du^1 du^2 du^3 (\hat{\Psi}_a^\dagger(\vec{u})\sigma^z_{ab}\hat{\Psi}_b(\vec{u})) \tag{1}$$

with σ^z_{ab} the Pauli matrix and $\hat{\Psi}_a(u_i)$ a field operator for a field theory of a Fermionic non-relativistic particle.

One wishes to assign a probability to the following properties that arise from the measurement of the spin components by the detector: a) the z-component of the spin of an electron in the region accessible to the detector and b) the physical position of the detector.

We denote the spin projectors associated with the detection of the electron by $\hat{P}^z_{\vec{x}}(\epsilon)$, with $\epsilon = \pm 1$ the electron spin along the z direction. We also represent as $\hat{P}^{\vec{x}_0}_{\vec{X}_0}$ the projectors associated to the measurement of the physical position of the center of the detector, $\vec{x}_0$ within a region $\Delta V_{\vec{X}_0}$ centered at $\vec{X}_0$, and we consider a continuous spectrum for $\vec{X}$. We also assume that the electron and measurement apparatus are independent systems and therefore their joint

* Present address: Department of Physics, University of California, Santa Barbara, CA 93106

density matrix is a tensor product,

$$\rho_{\text{total}} = \rho_A \otimes \rho_S, \tag{2}$$

In the above expression ρ_A is the density matrix associated with the measuring apparatus and ρ_S is the density matrix associated with the electron. This assumption is made in order to simplify calculations; a more realistic treatment taking into account the interaction is possible, but it would just add another source of noise that will contribute further to the effects we discus; some discussion of this point is in reference [4]).

We would like now to ask the question "what is the probability that the spin takes the value ϵ_0 given that the detector is located at certain physical point $\vec{X}_0$". Such question is embodied in the conditional probability,

$$\mathcal{P}(\epsilon_0|\vec{X} \in \Delta V_{\vec{X}_0}) = \lim_{L\to\infty} \frac{\int_L^L du^1 \int_L^L du^2 \int_L^L du^3\, \mathrm{Tr}(\hat{P}^z_{\vec{u}}(\epsilon_0)\hat{P}^{\vec{u}}_{\vec{X}_0}\rho\hat{P}^{\vec{u}}_{\vec{X}_0})}{\int_{-L}^L du^1 \int_{-L}^L du^2 \int_{-L}^L du^3\, \mathrm{Tr}\left(\hat{P}^{\vec{u}}_{\vec{X}_0}\rho\right)} \tag{3}$$

where we have used the properties of the projectors to rearrange the expression and the integrals over $\vec{u}$ in the right hand side are taken over the whole space. The reason for the integrals is that we do not know for what value of the fiducial coordinates $\vec{u}$ the rods will take the values $\vec{X}_0$. We now use the independence of the system and measurement apparatus to write,

$$\mathcal{P}(\epsilon_0|\vec{X} \in \Delta V_{\vec{X}_0}) = \lim_{L\to\infty} \frac{\int_{-L}^L du^1 \int_{-L}^L du^2 \int_{-L}^L du^3\, \mathrm{Tr}(\hat{P}^z_{\vec{u}}(\epsilon_0)\rho_S)\mathrm{Tr}(\hat{P}^{\vec{u}}_{X_0}\rho_A)}{\int_{-L}^L du^1 \int_{-L}^L du^2 \int_{-L}^L du^3\, \mathrm{Tr}(\hat{P}^{\vec{u}}_{\vec{X}_0}\rho_A)}. \tag{4}$$

The above expression may be written in terms of the probability of having measured $\vec{X}_0$ for a given value of $\vec{u}$, defined by

$$\mathcal{P}_{\vec{u}}(\vec{X}_0) = \frac{\mathrm{Tr}(\hat{P}^{\vec{u}}_{\vec{X}_0}\rho_A)}{\int_{-L}^L du^1 \int_{-L}^L du^2 \int_{-L}^L du^3\, \mathrm{Tr}(\hat{P}^{\vec{u}}_{\vec{X}_0}\rho_A)}. \tag{5}$$

that satisfies $\int du^1 du^2 du^3 \mathcal{P}_{\vec{u}}(X_0) = 1$. The resulting expression for the conditional probability is,

$$\mathcal{P}(\epsilon_0|\vec{X} \in \Delta V_{\vec{X}_0}) = \lim_{L\to\infty} \int_{-L}^L du^1 \int_{-L}^L du^2 \int_{-L}^L du^3\, \mathrm{Tr}(\hat{P}^z_{\vec{u}}(\epsilon_0)\rho_S)\mathcal{P}_{\vec{u}}(\vec{X}_0). \tag{6}$$

Due to the limitations in the measurements of lengths derived from the fact that accuracy in measurement requires expending energy, which in general relativity means distorting space-time, one cannot choose the measurement apparatus such that $\mathcal{P}$ takes the form of a Dirac delta. Some minimum width in the distribution is inevitable. This subject has a long history going back to Salecker and Wigner [5], continuing with the work of Ng and van Dam [6] and Lloyd and Ng [7]. Measurements of distances taking into account that energy must be spent in performing a measurement, are gravitationally limited. If one wishes to arbitrarily increase the accuracy of a measurement of distance eventually one hits a limit in that the energy densities involved would create a black hole. Although this provides a fundamental limit to the accuracy of measurement, it is obvious that in practice one will have considerably larger errors in measurement with more mundane origins. To model this one can consider that $\mathcal{P}$ is a Gaussian whose spread grows with the distance $|\vec{X}|$ between the origin of the measuring rods and the detector,

$$\mathcal{P}_{\vec{u}}(\vec{X}) = \frac{1}{(\pi D(\vec{X}))^{3/2}} \exp\frac{-(\vec{X}-\vec{u})^2}{D(\vec{X})} \tag{7}$$

with $D(\vec{X}) = {\ell_P}^{4/3}|\vec{X}|^{2/3}$ with ℓ_P given by Planck's length, as has been extensively discussed by [6] and in our previous papers [1]. (A Lorentz-covariant extension of these expressions, where essentially what happens is that $|\vec{X}|$ is replaced by the proper interval, has been discussed in [3]). As the dispersion grows with the distance to the reference point chosen from which to measure distances, the conditional probabilities are not invariant under translations. In fact if we consider a translation of the origin of the rods by a vector $\vec{a}$, then $\mathcal{P}_{\vec{u}}(\vec{X}_0) \neq \mathcal{P}_{\vec{u}+\vec{a}}(\vec{X}_0+\vec{a})$ and therefore

$$\mathcal{P}(\epsilon_0, \vec{X} \in \Delta V_{\vec{X}_0}) \neq \mathcal{P}(\epsilon_0, \vec{X} \in \Delta V_{\vec{X}_0+\vec{a}}) \tag{8}$$

This fact is at the core of the loss of entanglement we will discuss in this paper. This is a consequence from the fact that the uncertainty in measurements of space intervals is a function of distance and therefore translation invariance

is broken by the presence of an origin from which the measurements are obtained by means of a physical device, which we call a "real rod". In order to study the effects of the fundamental uncertainty in position on entangled systems we compare Bell's inequalities violations for a given position of the detectors as measured from different origins of the measuring rods coordinates.

Let us consider a two particle entangled system at a given time t_0 represented by the state,

$$|\Psi_{12} > |_{t_0} = \frac{1}{\sqrt{(2V(v_1)V(v_2)}}(|v_1,+;v_2,-\rangle + |v_1,-;v_2,+\rangle) \tag{9}$$

where the kets $|v_1,+;v_2,-\rangle$ represent states of two electrons, one with spin $+$ the other with spin $-$, the first one localized within the region v_1 (centered at $\vec{x}_1$), the second localized within the region v_2 and where $V(v_a)$ are the volumes of the regions v_a.

In order to measure the entanglement, we recall that a sufficient condition for the latter is the violation of Bell's inequalities. This phenomenon also exists in quantum field theory (see [8] for a discussion). We will show later on that a state that violates the inequalities maximally when measured locally, will violate them less strongly when the observer gets farther and farther away.

Schematically, the Bell inequalities work like this. One measures two quantities Q and R in a region centered at $\vec{x}_0$ and two other, S and T, in a region centered in $\vec{y}_0$. In our case we choose these quantities to be the spins in the region of the detector, as we defined above, with eigenvalues ± 1 or 0 (since we are in a quantum field theory, one has to allow for a no particle state). For definitiveness we will consider time-like localized measurements. That is, the detectors are devised such that they operate within a time window around t_0.

The Bell inequalities are obtained by assuming that the measured quantities depend on classical (hidden) variables and that:

a) the values of Q, R, S, T exist independently of the observation (realism),

b) Alice does not disturb the results of the measurements of Bob when making her measurements.

From these hypotheses one would get that,

$$< \hat{Q}(\vec{x}_0)\hat{S}(\vec{y}_0) + \hat{R}(\vec{x}_0)\hat{S}(\vec{y}_0) + \hat{R}(\vec{x}_0)\hat{T}(\vec{y}_0) - \hat{Q}(\vec{x}_0)\hat{T}(\vec{y}_0) > \leq 2. \tag{10}$$

Quantum mechanics, however, violates this inequality and predicts values that reach $2\sqrt{2}$.

We choose the operators as,

$$\begin{aligned}
\hat{Q}(\vec{x}_0) &= \hat{\sigma}^z(v_{\vec{x}_0}) = \int_{v_{\vec{x}_0}} du^1 du^2 du^3 \hat{\Psi}^\dagger_a(\vec{u})\sigma^z_{ab}\hat{\Psi}_b(\vec{u}) && (11)\\
\hat{R}(\vec{x}_0) &= \hat{\sigma}^x(v_{\vec{x}_0}) && (12)\\
\hat{S}(\vec{y}_0) &= \frac{-\hat{\sigma}^z(v_{\vec{y}_0}) + \hat{\sigma}^x(v_{\vec{y}_0})}{\sqrt{2}} && (13)\\
\hat{T}(\vec{y}_0) &= \frac{\hat{\sigma}^z(v_{\vec{y}_0}) + \hat{\sigma}^x(v_{\vec{y}_0})}{\sqrt{2}}. && (14)
\end{aligned}$$

One needs to assume that the regions $v_{\vec{x}}$ and $v_{\vec{y}}$ are spatially separated [8] when the detector at $\vec{x}$ is measuring the particle in $\vec{x}_1$ and the detector in $\vec{y}$ is measuring the particle in $\vec{x}_2$. If one assumes the regions are spherical, then their diameters should be smaller than $|\vec{x}_1 - \vec{x}_2|/2$. To simplify further calculations we will assume that the diameters are considerably smaller than the separation, as measured by local observers.

We wish to compute expectation values of products of these operators to construct Bell's inequality. We will consider now a more general situation with operators localized at points $\vec{X}$, $\vec{Y}$ by operational measurements of the position from an origin which is not necessarily close to the experimental arrangement. The expectation value of the products of these operators will be of the form,

$$\langle\Psi_{12}|\sigma^z(\vec{X})\sigma^z(\vec{Y})|\Psi_{12}\rangle = \int d^3x d^3y \langle\hat{\sigma}^z(v_{\vec{x}})\hat{\sigma}^z(v_{\vec{y}})\rangle \mathcal{P}_{\vec{x}}(\vec{X})\mathcal{P}_{\vec{y}}(\vec{Y}). \tag{15}$$

We start by computing

$$\hat{\sigma}^z(v_{\vec{y}})|\Psi_{12}\rangle = \frac{1}{\sqrt{2V(v_1)V(v_2)}}\left[|v_{\vec{y}} \cap v_1,+;v_2,-\rangle - |v_{\vec{y}} \cap v_1,-;v_2,+\rangle - |v_1,+;v_{\vec{y}} \cap v_2,-\rangle + |v_1,-;v_{\vec{y}} \cap v_2,+\rangle\right]. \tag{16}$$

To simplify things, we will assume that the volumes are equal and only depend on the position of the center. One then has,

$$\begin{aligned}\hat{\sigma}^z(v_{\vec{x}})\hat{\sigma}^z(v_{\vec{y}})|\Psi_{12}\rangle &= \frac{1}{\sqrt{2V(v_1)V(v_2)}}\left[|v_{\vec{x}}\cap v_{\vec{y}}\cap v_1,+;v_2,-\rangle - |v_{\vec{y}}\cap v_1,+;v_{\vec{x}}\cap v_2,-\rangle\right. \\ &+|v_{\vec{x}}\cap v_{\vec{y}}\cap v_1,-;v_2,+\rangle - |v_{\vec{y}}\cap v_1,-;v_{\vec{x}}\cap v_2,+\rangle \\ &-|v_{\vec{x}}\cap v_1,+;v_{\vec{y}}\cap v_2,-\rangle + |v_1,+;v_{\vec{x}}\cap v_{\vec{y}}\cap v_2,-\rangle \\ &\left.-|v_{\vec{x}}\cap v_1,-;v_{\vec{y}}\cap v_2,+\rangle + |v_1,-;v_{\vec{x}}\cap v_{\vec{y}}\cap v_2,+\rangle\right].\end{aligned} \tag{17}$$

Notice that if one takes $v_{\vec{x}} = v_1$ and $v_{\vec{y}} = v_2$ and $\mathcal{P} = \delta$ (local measurement) then, using equation (16) one has that $\vec{X} = \vec{x}_1$ and $\vec{Y} = \vec{y}_1$ and one is left only with the fifth and seventh term since v_1 and v_2 are disjoint and,

$$\langle\Psi_{12}|\hat{\sigma}^z(\vec{X})\hat{\sigma}^z(\vec{Y})|\Psi_{12}\rangle = -1, \tag{18}$$

which is the usual result in quantum mechanics.

In general, for arbitrary $v_{\vec{x}}$ and $v_{\vec{y}}$,

$$\begin{aligned}&\langle\Psi_{12}|\hat{\sigma}^z(\vec{X})\hat{\sigma}^z(\vec{Y})|\Psi_{12}\rangle = \int d^3x \int d^3y \frac{\mathcal{P}_{\vec{x}}(\vec{X})\mathcal{P}_{\vec{y}}(\vec{Y})}{2V(v_1)V(v_2)} \\ &\times\left[2V(v_2)V(v_{\vec{x}}\cap v_{\vec{y}}\cap v_1) - 2V(v_{\vec{y}}\cap v_1)V(v_{\vec{x}}\cap v_2) - 2V(v_{\vec{x}}\cap v_1)V(v_{\vec{y}}\cap v_2) + 2V(v_1)V(v_{\vec{x}}\cap v_{\vec{y}}\cap v_2)\right] \\ &= \int d^3x \int d^3y \mathcal{P}_{\vec{x}}(\vec{X})\mathcal{P}_{\vec{y}}(\vec{Y})\left[\frac{V(v_{\vec{x}}\cap v_{\vec{y}}\cap v_1)}{V(v_1)} + \frac{V(v_{\vec{x}}\cap v_{\vec{y}}\cap v_2)}{V(v_2)} - \frac{V(v_{\vec{x}}\cap v_1)V(v_{\vec{y}}\cap v_2)}{V(v_1)V(v_2)} - \frac{V(v_{\vec{x}}\cap v_2)V(v_{\vec{y}}\cap v_1)}{V(v_1)V(v_2)}\right].\end{aligned} \tag{19}$$

It is also the case that $\langle\Psi_{12}|\hat{\sigma}^z(\vec{X})\hat{\sigma}^x(\vec{Y})|\Psi_{12}\rangle = 0$ since after the action of these operators the spins are either are $++$ or $--$ and therefore orthogonal to the initial ones in $|\Psi_{12}\rangle$. For the x direction we have,

$$\begin{aligned}\langle\hat{\sigma}^x(\vec{X})\hat{\sigma}^x(\vec{Y})\rangle &= \int d^3x \int d^3y \mathcal{P}_{\vec{x}}(\vec{X})\mathcal{P}_{\vec{y}}(\vec{Y})\left[\frac{V(v_{\vec{x}}\cap v_{\vec{y}}\cap v_1)}{V(v_1)} + \frac{V(v_{\vec{x}}\cap v_1)V(v_{\vec{y}}\cap v_2)}{V(v_1)V(v_2)}\right. \\ &\left.+\frac{V(v_{\vec{x}}\cap v_2)V(v_{\vec{y}}\cap v_1)}{V(v_1)V(v_2)} + \frac{V(v_{\vec{x}}\cap v_{\vec{y}}\cap v_2)}{V(v_2)}\right].\end{aligned} \tag{20}$$

To set up Bell's inequalities we need the expectation values,

$$\langle\hat{Q}(\vec{X})\hat{S}(\vec{Y})\rangle = -\frac{1}{\sqrt{2}}\langle\hat{\sigma}^z(\vec{X})\hat{\sigma}^z(\vec{Y})\rangle \tag{21}$$

$$\langle\hat{R}(\vec{X})\hat{S}(\vec{Y})\rangle = \frac{1}{\sqrt{2}}\langle\hat{\sigma}^x(\vec{X})\hat{\sigma}^x(\vec{Y})\rangle \tag{22}$$

$$\langle\hat{R}(\vec{X})\hat{T}(\vec{Y})\rangle = \frac{1}{\sqrt{2}}\langle\hat{\sigma}^x(\vec{X})\hat{\sigma}^x(\vec{Y})\rangle \tag{23}$$

$$\langle\hat{Q}(\vec{X})\hat{T}(\vec{Y})\rangle = \frac{1}{\sqrt{2}}\langle\hat{\sigma}^z(\vec{X})\hat{\sigma}^z(\vec{Y})\rangle, \tag{24}$$

so one has

$$\langle\hat{Q}\hat{S} + \hat{R}\hat{S} + \hat{R}\hat{T} - \hat{Q}\hat{T}\rangle = \frac{4}{\sqrt{2}}\int d^3x \int d^3y \mathcal{P}_{\vec{x}}(\vec{X})\mathcal{P}_{\vec{y}}(\vec{Y})\left[\frac{V(v_{\vec{x}}\cap v_1)V(v_{\vec{y}}\cap v_2) + V(v_{\vec{x}}\cap v_2)V(v_{\vec{y}}\cap v_1)}{V(v_1)V(v_2)}\right] \tag{25}$$

Therefore for $\mathcal{P} = \delta$ and $v_{\vec{x}} = v_1$ and $v_{\vec{y}} = v_2$ one has that

$$\langle\hat{Q}\hat{S} + \hat{R}\hat{S} + \hat{R}\hat{T} - \hat{Q}\hat{T}\rangle = 2\sqrt{2}, \tag{26}$$

as in standard quantum mechanics, leading to a maximal violation of Bell's inequality. This result corresponds to a local measurement where the origin of measurement is close to the particles and the particles are close to each other, so one does not have to worry about the dispersion we are studying.

Let us now consider the situation where these hypotheses do not hold anymore. That is, we will consider x_1 and x_2 to be close to each other but we will measure the position of the detectors from an origin that is at a large distance a from the detectors. So we will now consider functions $\hat{Q}(\vec{X})$ with $\vec{X} \sim a$, etc. Let us take $\vec{x}_1$ and $\vec{x}_2$ to be the position

of v_1 and v_2 (measured locally). When measuring the system from a distance $\vec{X}$ and $\vec{Y}$ such that $\sqrt{D(\vec{a})} \sim |\vec{x}_2 - \vec{x}_1|$ (with $D(\vec{a}) = \ell_P{}^{4/3}|\vec{a}|^{2/3}$ as before), we will have to include in the average of equation (25) regions where $v_x \cap v_1$ and $v_x \cap v_2$ (and similarly for v_y) are empty. This immediately implies that the right hand side of Bell's inequality (26) will be smaller than $2\sqrt{2}$. It should also be noted that when measuring the system from a distance $\vec{a}$ such that $\sqrt{D(\vec{a})} \sim r(v_{\vec{x}})$ or that $\sqrt{D(\vec{a})} \sim r(v_{\vec{y}})$ where $r(v_{\vec{x}})$ is the characteristic size of the region $v(\vec{x})$ and similarly for $\vec{y}$, the effect will also be large. This is due to the fact that we will have to include regions where $v_{\vec{x}} \cap v_1$ and $v_{\vec{y}} \cap v_2$ are empty. In most practical applications this will be in fact the dominant effect. However, this effect can always be diminished by increasing the size of the detectors.

We could introduce operators with $v_{\vec{x}}$ and $v_{\vec{y}}$ much lager than v_1 and v_2 and in that case the inequality would still be violated if we only take into account the previous arguments, although it wont achieve the value $2\sqrt{2}$. The combined effect of both contributions is again the elimination of the correlation between both particles. In other words the entanglement observed by different observers is different and the degree of entanglement diminishes as the system is observed from further and further away. In this sense we can say that the entanglement *is not* translation invariant. It is known that the entanglement of spins is not Lorentz invariant [9]. In this case this is an additional effect.

The effect we are considering, for a system of two entangled particles, will always be very small for measurements done close to the particles (that we assume are not far apart from each other), which are the usual kinds of measurements considered for entangled systems. Therefore, even though in practice there is loss of entanglement due to a translation to a different observational point, the entanglement can be always recovered by choosing a local observer.

The effect becomes more interesting when one considers more than two entangled particles. There one can have more favorable configurations. Consider a system of four particles consisting of two pairs of particles close to each other within each pair, whereas the pairs are far away from each other. Let us call the separation of the particles within each pair δ and the separation between pairs d. We then have that $d \gg \delta$. In this case our effect will be of increasing importance if $D(d) \sim \delta^2$. The effect would then go as,

$$\exp\left[-\left(\frac{\delta}{d^{1/3}\ell_{\rm Planck}^{2/3}}\right)^2\right]. \tag{27}$$

So for instance, if one considered δ of the order of nuclear distances, one would require d to be of the order of millions of kilometers for the effect to be of the order of 10^{-6}. This cannot be completely discarded in the context of experiments involving interferometry in space.

We have shown that the use of realistic measuring rods in quantum mechanics inevitably leads to a loss of entanglement. The effect depends on the distance from the measured system to the origin chosen and goes as the distance to the one third power. The magnitude of the effect is very small for everyday experiments, but could put limitations to teleportation over astronomical distances.

Another interesting implication of the effect is for the measurement problem in quantum mechanics [10, 11, 12]. In the usual treatment of the measurement problem, one considers decoherence due to interaction with the environment. In these effects one assumes that simultaneous measurements of the system, the measurement apparatus and the environment are impossible due to practical limitations having to do with the large number of degrees of freedom of the environment. The existence of the loss of entanglement we discuss would add fundamental (and not only practical) limitations to the simultaneous measurement of system, apparatus and environment. As such, it could bypass usual objections that suggest that at least in principle such measurements would be possible. Our effect provides a definite reason for the passage from the quantum to the classical world that cannot be overridden even in principle.

This work was supported in part by grant NSF-PHY-0554793, funds of the Hearne Institute for Theoretical Physics, FQXi, CCT-LSU and Pedeciba (Uruguay). RAP is supported in part by DOE contracts DOE-ER-40682-143 and DEAC02-6CH03000.

---

[1] See R. Gambini, R. Porto and J. Pullin, Gen. Rel. Grav. **39**, 1143 (2007) [arXiv:gr-qc/0603090] and references therein.
[2] R. Gambini, R. A. Porto and J. Pullin, Phys. Rev. Lett. **93**, 240401 (2004) [arXiv:hep-th/0406260].
[3] R. Gambini, R. A. Porto and J. Pullin, Int. J. Mod. Phys. D **15**, 2181 (2006) [arXiv:gr-qc/0611148].
[4] S. D. Bartlett, T. Rudolph, R. W. Spekkens, and P. S. Turner, New J. Phys. **8**, 58 (2006) [quant-ph/0602069]; D. Poulin and J. Yard, New J. Phys **9**, 156 (2007) [quant-ph/0612126].
[5] E. Wigner, Rev. Mod. Phys. **29**, 255 (1957); H. Salecker, E. Wigner, Phys. Rev. **109**, 571 (1958).
[6] Y. J. Ng and H. van Dam, Annals N. Y. Acad. Sci. **755**, 579 (1995) [arXiv:hep-th/9406110]; Mod. Phys. Lett. A **9**, 335 (1994).

[7] S. Lloyd, Y. J. Ng, Scientific American, **281**, 52 (2004).
[8] A. Peres and D. R. Terno, Rev. Mod. Phys. **76**, 93 (2004) [arXiv:quant-ph/0212023]; L. J. Landau, Phys. Lett. **A120**, 54 (1987); S. J. Summers and R. Werner, J. Math. Phys **28**, 2440 and 2447 (1987).
[9] A. Peres, P. F. Scudo and D. R. Terno, Phys. Rev. Lett. **88** (2002) 230402 [arXiv:quant-ph/0203033].
[10] B. d'Espagnat "Veiled reality", Addison Wesley, New York (1995).
[11] W. Zurek, Phys. Rev. **D26**, 1862 (1982).
[12] R. Gambini and J. Pullin, Found. Phys. **37**, 1074 (2007) [arXiv:quant-ph/0608243].